Homework 3. This homework is listed as problem and then solution.

Starting with xl=-5 and xr=11, perform the bisection method for 4 iterations on f(x)=x4+x2

f’(x)=4x3+2x

xleft xmid xright f’(xmid)

-5 3 11 114

-5 -1 3 -6

-1 1 3 6

-1 0 1 0

Since f’(xmid) =0, a local optimal solution occurs at 0. If not I would have reported a local optimal exists between –1 and 1 and my best guess is to let x=0.

Starting with xl=-5 and xr=11, perform the bisection method for 4 iterations on f(x)=4x4-x2+5

f’(x)=16x3-2x

xleft xmid xright f’(xmid)

-5 3 11 426

-5 -1 3 -14

-1 1 3 14

-1 0 1 0

Since f’(xmid) =0, a local optimal solution occurs at 0. If not I would have reported a local optimal exists between –1 and 1 and my best guess is to let x=0.

Starting with xl=-5 and xr=11, perform the golden search method for 4 iterations for

f(x)=x4+x2.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| xleft | xmid1 | xmid2 | xright | f(xleft) | f(xmid1) | f(xmid2) | f(xright) |
| -5 | 1.112 | 4.888 | 11 | 650 | 2.765585 | 594.7462 | 14762 |
| -5 | -1.22278 | 1.110784 | 4.888 | 650 | 3.730826 | 2.756205 | 594.7462 |
| -1.22278 | 1.111535 | 2.553681 | 4.888 | 3.730826 | 2.761999 | 49.04843 | 594.7462 |
| -1.22278 | 0.217273 | 1.106943 | 2.547 | 3.730826 | 0.049436 | 2.726735 | 48.57109 |

I know a local optimal solution exists between –1.22 and 1.1. My best guess is x=.217 with a objective value of .05.

Starting with xl=-5 and xr=11, perform the golden search method for 4 iterations on f(x)=4x4-x2+5

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| xleft | xmid1 | xmid2 | xright | f(xleft) | f(xmid1) | f(xmid2) | f(xright) |
| -5 | 1.112 | 4.888 | 11 | 2480 | 9.87962 | 2264.522 | 58448 |
| -5 | -1.22278 | 1.110784 | 4.888 | 2480 | 12.4473 | 9.855614 | 2264.522 |
| -1.22278 | 1.111535 | 2.553681 | 4.888 | 12.4473 | 9.87044 | 168.5873 | 2264.522 |
| -1.22278 | 0.217273 | 1.106943 | 2.547 | 12.4473 | 4.961707 | 9.780332 | 166.8483 |

I know a local optimal solution exists between –1.22 and 1.1. My best guess is x=.217 with a objective value of 4.96.

From a starting points of 0 and 1, perform the intial search algorithm to determine an optimal region for x2-10x+25.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| xleft | xmid | xright | f(xleft) | f(xmid1) | f(xright) |
| 0 | 0.5 | 1 | 25 | 20.25 | 16 |
| 0 | 1 | 2 | 25 | 16 | 9 |
| 0 | 2 | 4 | 25 | 9 | 1 |
| 0 | 4 | 8 | 25 | 1 | 9 |

I know that there exists a local optimal between 0 and 8. If I wanted to find it I would use xleft=0 and xright =8 in either bisection or the golden search methods.

a starting points of 0 and 1, perform the intial search algorithm to determine an optimal region for x2-.2x+.04.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| xleft | xmid | xright | f(xleft) | f(xmid1) | f(xright) |
| 0 | 0.5 | 1 | 0.04 | 0.19 | 0.84 |
| 0 | 0.25 | 0.5 | 0.04 | 0.0525 | 0.19 |
| 0 | 0.125 | 0.25 | 0.04 | 0.030625 | 0.0525 |

I know that there exists a local optimal between 0 and .25. If I wanted to find it I would use xleft=0 and xright =.25 in either bisection or the golden search methods.

Starting with xl=-5 and xr=11, perform the Fibonicci search method starting with n=6 (Fn=8)

f(x)=x4+x2.

xmid1 = -5 + 3/8\*(11- -5) = 1

xmid2 = -5 + 5/8\*(11 - -5) = 5

\*\* Notice that the second iteration is a tie. Can either side be thrown away? Checking to see xmid1+ε verifies that the left side can be thrown away.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| xleft | xmid1 | xmid2 | xright | f(xleft) | f(xmid1) | f(xmid2) | f(xright) |
| -5 | 1 | 5 | 11 | 650 | 2 | 650 | 14762 |
| -5 | -1 | 1 | 5 | 650 | 2 | 2 | 650 |
| -1 | 1 | 3 | 5 | 2 | 2 | 90 | 650 |
| -1 | 1 | 1 | 3 | 2 | 2 | 2 | 90 |

Notice how the Fibonicci search ends in a single point of 1.

Perform two iterations of the gradient search method on f(x,y)= x2+4xy+2y2+2x+2y. Use (0,0) as a starting point.

The gradient of f is |2x+4y+2|

|4x+4y+2|

d1= the negative of the gradient of f at the point (0,0) = -|2|

|2|

So

x1=(0,0) +λ(-2,-2) = (-2λ,-2λ).

Plugging this into f(x,y) results in

f(λ)=4λ2+16λ2+8λ2-4λ-4λ=28λ2-8λ

Taking the deriviative and setting it equal to 0 to find the min results in

f’(λ)=56λ-8=0

λ=1/7

So x1=(-2/7,-2/7).

d2= the negative of the gradient of f at the point (-2/7,-2/7) = -|2/7 |

|-2/7|

So

x1=(-2/7,-2/7) +λ(-2/7,2/7) = (-2/7-2λ/7, -2/7+2λ/7).

Plugging this into f(x,y) results in the following simplified expression.

f(λ)=-4/49λ2 –8/49λ-28/49

Taking the deriviative and setting it equal to 0 to find the min results in

f’(λ)= -8/49λ –8/49 =0

λ=-1

So x2=(0,-4/7).

1. Perform 2 iterations of Newton’s method on f(x,y)=x4+4x2y+4y2+2x+2y starting at (1,0).

X0=(1,0)

The gradient of f(x)= | 4x3+8xy+2 |

| 4x2+8y+2 |

The Hessian of f(x) is | 12x2 8x | = at (1,0) it is |12 8 |

| 8x 8 | |8 8 |

The inverse of the Hessian is | 1/4 1/4 |

| 1/4 -3/8 |

The gradient at (1,0) is | 6 |

| 6 |

X1= | 1 | - |1/4 -1/4 | | 6 | = |1 | - | 0 |

| 0 | |-1/4 3/8 | | 6 | |0 | | ¾ |

So X1= | 1 |

|-3/4 |

The gradient at (1,-3/4) is | 0 |

| 0 |

x2= | 1 |

No change occurred in the solution, so a local optimal candidate point is

x2 = | 1 |

|-3/4 |

Perform 1 complete iteration of the cyclical coordinate search and then find the Hooke Jeaves direction

f(x1,x2) = x12-4x1 +2x22 starting at (1,1)

x1 = (1,1)+λ (1,0) =

f=(1+λ)2 -4(1+λ) +2

The derivative is

2(1+λ) -4 = 0, so λ=-1.

So x1 = (0,1)

x2 = (0,1)+λ (0,1)

f= 2(1+λ)2= taking the derivative yields

4λ +4, so λ=-1,

The new point is

x2=(0,0)

Hookes and Jeeves direction is (1,1) –(0,0) = (1,1).

x3 = (0,0)+λ (1,1)

f = (λ)2 +-4λ +2λ2

Taking the derivative yields

2λ-4 +2+4λ =0 so λ=2/3,

The new point is

x3=(2/3,2/3)